ROUGH SET ON GENERALIZED COVERING APPROXIMATION SPACES

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ABSTRACT :

An approach to capture impreciseness, Pawlak introduced the notion of rough sets, which is an excellent tool to capture indiscernibility of objects. An equivalence relation is the simplest formulization of the indiscernibility. The basic assumption of rough set theory is that human knowledge about a universe depends upon their capability to classify its objects. Classifications of a universe and equivalence relations defined on the universe are known to be interchangeable notions. So, for mathematical point of view, equivalence relations are considered to define rough set. An inexact set (A rough set) is represented by a pair of exact sets called the lower approximation and upper approximation of the set. The lower approximation of a rough set comprises of those elements of the universe which can be said to belong to it definitely with the available knowledge. The upper approximation comprises of those elements which are possibly in the set with respect to the available information (knowledge). In this note we introduced Covering Based Rough Sets which is an extension to the traditional (Z. Pawlak) Rough Sets.

Keywords : Lower approximation, upper approximation, indiscernibility, rough sets, covering.

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6.1 INTRODUCTION

Undoubtedly Z.Pawalak's introduction of the rough set theory is born out of his long expedition in the forest chaos originated from insufficient and incomplete information system .This inspires him to search for a comprehensible theory for classification , concept formation and data analysis. The new theory upholds the mathematical approach for the study of indiscernibility of objects. Indiscernibility of object refers though the granularity of knowledge that effects the definition of universe of discourse. Indiscernibility may be described by equivalence relations. Defining the indiscernibility of object includes the objects of universe represented by a set of attributes, based on their attribute values.

The principle of rough set theory enables to conceptualize , organize and analyze different types of data in data mining . In addition to this, the theory is vary much useful for dealing with uncertain and vague knowledge in the information systems. The extensive application of rough set models are found in Process control, Economics, Medical diagnosis, Biochemistry, Environmental science, Biology, Psychology, Conflict analysis and other area of knowledge ([1], [6], [8], [9], [10],[12], [15], [16], [22]).

Most of the existing methods for modeling, reasoning and computing are crisp, deterministic and precise in character. However, the situations in our day to day life are hardly crisp and deterministic and precise. For presenting a complete and realistic description of a system one would certainly require a more detailed data to know , organize as well as comprehend the vast information system. This new observation extends the concept of crisp set for modeling the imprecise data by enriching the modeling power. In order to face such trying situations, necessarily, researchers have developed many advance techniques such as fuzzy set theory ([18]) Dempster - Shafer theory of evidence ([10]), rough set theory ([4]), computing with words ([13], [19], [20], [21]) computational theory for linguistic dynamic systems ([14]) and granular computing ([2]).

More over to take control of impreciseness, Pawlak introduced the idea of rough sets as a brilliant exposition to study indiscernibility of objects. An equivalence relation is the simplest formulization of the indiscernibility. The knowledge of human beings about a universe depends upon their faculties to classify its objects, granularity crops up the basic assumption of rough set theory. Classifications of a universe and equivalence relations defined on the universe are known to be interchangeable notions. So, for mathematical point of view, equivalence relations are very essential to define rough set. An inexact set (A rough set) is represented by a pair of exact sets called the lower approximation and upper approximation of the set. The lower approximation of a rough set comprises of those elements of the universe which can be said to belong to it definitely with the available knowledge. The upper approximation comprises of those elements which are possibly in the set with respect to the available information (knowledge).

The rough set theory, proposed by Pawlak, is termed as basic rough set theory and it has been extended in many directions. Covering based rough set is one of the extensions of the basic rough set theory. In recent past W. Zhu and F.Y. Wang (2006 [23], [24] [25]) have proposed four types of covering rough sets in which only one lower approximation and four different versions of upper approximations for such rough sets, several properties of these different types of covering rough sets are derived and analyzed. This paper proposes a new type of covering based rough set which generalizes both lower approximation and upper approximation operators.

2. Rough set approximation operators

Let U be finite set of objects, called the universe of discourse, and $U \neq \varphi$, and R be an be equivalence relation (knowledge or information) over U called an indiscernibility relation. By U/R we denote the family of all equivalence classes of R (or classification of U) referred to as categories or concepts of R and $[x]_R$, an equivalence class of x in R, denotes a category in R containing an element $x \in U$. We say an ordered pair A = (U, R), an approximation space and a relational system **K** = (U, **R**) is called a knowledge base, where U is a universe and **R** is family of equivalence relations defined on U.

For any set $X \subseteq U$, the lower approximation of X in the approximation space A under the indiscernibility relation R be defined by

(i) $\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}$

And an upper approximation of X in A under R be defined by

(ii) $\overline{R}(X) = \{x \in U : [x]_R \cap X \neq \varphi\}$

A set $X \subseteq U$ is called rough with respect to R if and only if $\underline{R}(X) \neq \overline{R}(X)$ and X is called definable (or exact) with respect to R if and only if $\underline{R}(X) = \overline{R}(X)$. The border line region of X with respect to R is denoted by $BN_R(X)$ and is given by $BN_R(X) = \overline{R}(X) - \underline{R}(X)$.

For an element $x \in U$, x is certainly in X under the equivalence relation R if and only if $x \in \underline{R}(X)$ and x is possible in X under R if and only if $x \in \overline{R}(X)$. The borderline region under R is the undecidable area of the universe. We say X is rough with respect to R if and only if $\underline{R}(X) \neq \overline{R}(X)$, which is equivalent to $BN_R(X) \neq \varphi$, otherwise X is said to be R definable, that is, $BN_R(X) = \varphi$.

The discussion proposes four different types of covering based rough sets. These are basically due to four different types of upper approximations introduced where as the lower approximations are all equal.

3. Rough sets on Covering approximation space

The following definitions are essentially required to define the covering based rough sets.

Definition 3.1 : Let U be a universe of discourse and C be a family of nonempty subset of U. C is called a cover of U if $\cup$ C = U.

We call (U, C) the covering approximation space and the covering C is called the family of approximation sets.

It is clear that a partition of U is certainly a covering of U, so the concept of a covering is an extension of a partition.

Definition 3.1 : Let (U, C) be an approximation space and x be any element of U. then the family.

$Md(x) = \{K \in C : x \in K \wedge \forall S \in C[x \in S \wedge S \subseteq K \Rightarrow K = S]\}$ is called the minimal description of the object x.

In order to describe an object we need only the essential characteristics related to this object. This is the purpose of the minimal description concept.

Definition 3.3 : For any set $X \subseteq U$ the family of sets.

$C_*(X) = \{K \in C : K \subseteq X\}$ is called bottom approximation of the set X.

Definition 3.4 : The set $\underline{X} = \cup C_*(X)$ is called lower approximation of the set X.

Definition 3.5 : [24] Let U be a non empty finite universe and C be a covering of U. The covering upper approximation of first type of $X \subseteq U$ be defined by $FH(X) = \underline{X} \cup \{Md(x) : x \in X - \underline{X}\}$. The set X is called first type covering based rough if $\underline{X} \neq FH(X)$ otherwise X is called exact set. The F-boundary of X be given by $BN_F(X) = FH(X) - \underline{X}$ is called as boundary region or border line region of X of first type covering.

Definition 3.6 : [25] Let (U, C) be a covering approximation space. For approximation set $X \subseteq U$, the covering upper approximation of second type of X be defined by $SH(X) = \cup\{K : K \in C, K \cap X \neq \varphi\}$. The set X is called second type covering based rough when $\underline{X} \neq SH(X)$, otherwise X is called exact set.

The S-Boundary of X be given by $BN_S(X) = SH(X) - \underline{X}$ is known as the boundary region of X of second type covering.

Definition 3.7 : [12] Let (U, C) be a covering approximation space, for approximation set $X \subseteq U$, the covering upper approximation of third type of X be defined by $TH(X) = \cup\{Md(x) : x \in X\}$.

The set X is called third type covering based rough if $\underline{X} \neq TH(X)$, otherwise X is called exact set of third type covering.

The T-boundary of X be given by $BN_T(X) = TH(X) - \underline{X}$ is known as the boundary region of X of third type covering.

Definition 3.8 : [26] Let (U, C) be a covering approximation space. For $X \subseteq U$, the covering upper approximation of fourth type of X be defined by $\overline{X} = \underline{X} \cup \{K : K \in C \text{ and } K \cap (X - \underline{X}) \neq \varphi\}$.

The set X is called fourth type of covering based rough if $\underline{X} \neq \overline{X}$ otherwise X is called exact set of fourth type of covering.

The fourth boundary of X be given by $BN_{fourth}(X) = \overline{X} - \underline{X}$ which is called as the boundary region of X of fourth type covering.

We use the notations FL (X), SL(X), TL(X) and $\underline{X}$ for the covering lower approximation of first type, second type, third type and fourth type respectively. We see that the lower approximations are all same, that is,

$FL(X) = SL(X) = TL(X) = \underline{X} = \cup\{K \in C : K \subseteq X\}$. We note that $\underline{X}$ = X for any $X \subset U$, if and only if X is the union of some elements in C.

If C is a partition of U, then $\underline{R}(X) = \underline{X}$ and $\overline{R}(X) = FH(X) = SH(X) = TH(X) = \overline{X}$ for all $X \subseteq U$.

The main objective of this note is to define new type covering based rough set.

4. Generalized Covering Based Rough set

Using the notations given earlier we write X' be the complement of X in U, $X' = -X = U - X$

Definition 4.1 : Let (U, C) be a covering approximation space. For any subset $X \subseteq U$, the covering lower approximation of X be defined by $X_* = \underline{X} = U\, C_*(X)$ and the covering upper approximation of X be defined by

$X^* = \cap\{K' : K \subseteq X' \text{ and } K \in C\}$.

The set X is called new type covering based rough when $X_* \neq X^*$, otherwise X is called an exact set. The boundary of X denoted by $BN_C(X) = X^* - X_*$, is called as the borderline region of X of the new type covering C.

We find the following propositions as :

Proposition 4.1 : $X_* = X$ if and only if X is the union of some elements of C and also $X^* = X$ of and only if X' is the union of some elements of C.

If C is a partition of U, then it becomes $X_* = \underline{R}X$ and $X^* = \overline{R}X$.

Proposition 4.2 : The new type covering lower and upper approximations have the following properties, for $X, Y \subseteq U$.

4.1 $U_* = U$, $U^* = U$

4.2 $\phi_* = \phi$, $\phi^* = \phi$

4.3 $X_* \subseteq X \subseteq X^*$

4.4 $(X_*)_* = X_*$, $(X^*)^* = X^*$

4.5 $X \subset C \Rightarrow X^* \subseteq y^*$ and $X_* \subseteq Y_*$

4.6 $(X \cap Y)^* \subseteq X^* \cap Y^*$

4.7 $(X \cup Y)^* \supseteq X^* \cup Y^*$

Example 1: Let U={a, b, c, d}and C={{a, b}, {b, c}, {b, c, d}, {d}} be a cover of U. for X = {a, c}, $\underline{X} = X_* = \phi$ and FH (X) = {a, b, c}, SF (X) = {a, b, c, d}, TH (X) = {a, b, c}, $\overline{X} = \{a,b,c,d\}, X^* = \{a,b,c\}$ for Y = {c, d}, we find $Y_* = \underline{Y} = \{d\}$ and FH(Y) = {b, c, d}, SH(Y) = {b, c, d} = TH (Y) = $\overline{Y}$, but $Y^* = \{c,d\}$.

Example 2 : Let U = {a, b, c, d, e} be the universe and C = {{a, d}, {b, c, d}, {c, d, e}, {a, d, e}} be a cover of U. Let X = {b, d}, then $\underline{X} = X_* = \phi$ and FH (X) = SH (X) = TH (X) = $\overline{X} = X^*$. for Y = {e} we get $\underline{Y} = Y_* = \phi$ and FH (Y) = {a, c, d, e} = SH (Y) = TH (Y) $= \overline{Y}$ *and* $Y^* = \{e\}$. Let Z = {a, d}, then $\underline{Z} = Z^* = \{a,d\}$ *and* $FH(Z) = \{a,d\}$, SH(Z)={a,b,c,d,e} = TH (Z), $\overline{Z} = \{a,d\}, Z^* = U.$

Let C_1 = {{a, d},{b, c, d}, {c, d}, {a, d, e}}be another covering of U and for P = {a, c}, $\underline{P} = P_* = \phi$ and FH (P) = {a, c, d}, SH (P) = U, TH (P) = {a, c, d, }, $\overline{P} = P_* = U.$

Example 3: Let U = {a, b, c, d, e, f} and C = {{a, b}, {a, d}, {a, c}, {b, c}, {b, d}, {b, f}, {c, d}, {e, f}} be a cover of U, for X = {b, d} we get $\underline{X} = \{b,d\} = X_*$ and FH (X) = {b, d} , SH (X) = {a, b, c, d, f}, TH (X) = {a, b, c, d, f}, $\overline{X} = \{b,d\}, X^* = \{b,d\}$, Let Y = {d, e, f}, then $\underline{Y} = Y_* = \{e,f\}, FH(Y) = SH(Y) = TH(Y) = \overline{Y} = U$ and $Y^* = \{d,e,f\}$

The upper approximation of any set $X \subseteq U$, for the covering C, contains no element of the lower approximation of the complement of X in U, that is, X* contains no element of ($(X')_*$. So that, for the approximation space (U, C),

$X_* = UC_*(X) = ((X')^*)'$ and $X^* = \cap\{K' : K \subseteq X',\ K \in C\} = U - (X')^*$

and hence $(X')_* = (X^*)'$ and $(X')^* = (X_*)'$

Proposition 4.3 Let (U, C) be a covering approximation space and let $X, Y \subseteq U$, then

(i) (XUY)*=X* U Y* if and only if ($(X \cap Y)_* = X_* \cap Y_*$

(ii) $(X \cap Y)^* = X^* \cap Y^*$ if and only if $(X \cup Y)_* = X_* \cup Y_*$

Proof : Suppose that $(X \cap Y)_* = X_* \cap Y_*$ for any sets $X, Y \subseteq U$.

Now,

$$(X \cup Y)^* = \cup - \left((X \cup Y)'\right)_*$$
$$= \cup - (X' \cap Y')_*$$
$$= \cup - ((X')_* \cap (Y')_*), \text{ from hypothesis}$$
$$= [\cup - (X')_*] \cup [\cup - (Y')_*]$$
$$= X^* \cup Y^*$$

Conversely,

$$X_* \cap Y_* = \left((X_*)'\right)' \cap \left((Y_*)'\right)'$$
$$= \left[(X_*)' \cup (Y_*)'\right]'$$
$$= \left((X')^* \cup (Y')^*\right)'$$
$$= \left((X' \cup Y')^*\right)', \text{ from hypothesis}$$

$$=\left(\left(\left(x\cap Y\right)'\right)^{*}\right)'$$

$$=\left(X\cap Y\right)_{*}$$

hence (i) is proved

A set $X\subseteq U$ is said to be upper exact if $\overline{R}(x)=X$ (or $X^{*}=X$ for the covering approximation space (U, C)) and X is said to be lower exact if $\underline{R}(X)=X$(or $X_{*}=X$ for the covering approximation space (U, C)).

Theorem 4.2 : Let (U, C) be a covering approximation space, then $X\subseteq X^{*}\subseteq SH(X)$ for $X\subseteq U$

Proof : Let x be an element of X^{*}, then $x\in\bigcap\{K':K\in C \text{ and } K\subseteq X'\}$

$\Leftrightarrow x\in K' \text{ for } K\in C \text{ and for each } K\subseteq X'$

$\Leftrightarrow x\in K \text{ for } K\in C \text{ and for each } K\subseteq X'$

$\Rightarrow$ *For all* $x\in U$ these exists $M\in C$ such that $x\in M$ and $M\cap X\neq\phi$.

$\Leftrightarrow x\in SH(X)$

This proves the theorem.

Theorem 4.3 Let (U, C) be a covering approximation space and let $X\subseteq U$. If X' is the union of some elements of C then $X^{*}\subseteq FH(X), SH(X), TH(X) \text{ and } \overline{X}.$

Proof : when X' is the union of some elements of C, X be upper exact , that is $X=X^{*}$ and hence the theorem.

Let U be a nonempty finite universe and C be a covering of U, then the maximal description of $X\in U$ on the covering C be defined by

$$M_{x}(X)=\{P\in C \mid x\in P\wedge\forall Y\in C[x\in y\wedge P\subseteq Y \text{ implies } P=Y\}.$$

The objective of the maximal description of an element x is to find all the possible characteristics related to this elements x.

5. Dependency

Let U be nonempty finite universe and let C_1, C_2 be two coverings of U. we say $C_1\subseteq C_2$ if and only if for every $K\in C_1$ there exists at least one $M\in C_2$ such that $K\subseteq M$.

Definition 5.1 Let C_1, C_2 be two coverings of U. Then the covering C_1 depends upon C_2 denoted by $C_2\Rightarrow C_1$ if and only if $C_2\subseteq C_1$ and $BN_{c_2}(X)\subseteq BN_{c_1}(X)$ for all $X\subseteq U$.

The coverings C_1 *and* C_2 are independent if and only if neither $C_1\Rightarrow C_2$ nor $C_2\Rightarrow C_1$ and are equivalent if and only if $C_1\Rightarrow C_2$ and $C_2\Rightarrow C_1$.

Now we define a covering C for the two given coverings C_1 and C_2 by $C=\{P\subseteq U \mid P\in C_1 \text{ or } P\in C_2\}$

The union and intersection for the two coverings C_1 and C_2 be defined by

$$C_1 \text{ and } C_2 = \bigcup_{x\in U} M_x(x) \text{ on the covering C}$$

$$=\bigcup_{x\in U}\{K\in C \mid x\in K\wedge[\forall Y\in C, x\in Y\wedge K\subseteq Y \text{ implies } K=Y]\} \text{ and}$$

$$C_1\cap C_2=\bigcup_{x\in U} Md(x) \text{ on the covering C}$$

$$=\bigcup_{x\in U}\{K\in E: x\in K\wedge[\forall S\in C, x\in S\wedge S\subseteq K \text{ implies } K=S]\}.$$

clearing $C_1\cup C_2$ and $C_1\cap C_2$ be two coverings of $\cup$.

Proposition 5.1 Let U be a non empty finite universe an let $(U,C_1),(U,C_2)$ *and* (U,C_3) be three covering approximation spaces where C_1, C_2 and C_3 be three different coverings of U. Then $C_2\Rightarrow C_1$ and implies $C_3\Rightarrow C_2$ and $C_3\Rightarrow C_1$.

Prof : Suppose that $C_2 \Rightarrow C_1$ and $C_3 \Rightarrow C_2$.

Then for any subset $X \subseteq U, BN_{c_2}(X) \subseteq BN_{c_1}(X)$ and $BN_{c_3}(X) \subseteq BN_{c_2}(X)$ and also $C_2 \subseteq C_1, C_3 = C_2$ so that $BN_{c_3}(X) \subseteq BN_{c_1}(X)$ and hence the Proposition.

Proposition 5.2 Let U be non empty finite universe and C_1, C_2 be two coverings of U, then

(i) $C_1 \cap C_2 \Rightarrow C_1$ provided $C_1 \cap C_2 \subseteq C_1$

(ii) $C_1 \cap C_2 \Rightarrow C_1 \cup C_2$

Example 4 : Let $U = \{a, b, c\}$ be the universe and $C_1 = \{\{a\}, \{b, c\}, \{c, a\}\}, C_2 = \{\{a\}, \{a, b\}, \{c\}\}$ to coverings on U. Now we find a covering C of U from the coverings C_1 and C_2 by

$C = \{\{a\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$ *then*

$C_1 \cup C_2 = \{\{a, b\}, \{b, c\}, \{c, a\}\}$ and

$C_1 \cap C_2 = \{\{a,\}, \{c\}, \{a, b\}, \{b, c\}\}$, so that

$C_1, C_2 \subseteq C_1 \cup C_2$ but $C_1 \cap C_2 \not\subset C_1, C_1 \cap C_2 \not\subset C_2$. Let C_3 be another covering of U given by

$C_3 = \{\{a\}, \{b\}, \{c\}\}$. *Clearly* $C_3 \subset C_1$ and for any subset $X \subseteq U$, $BN_{c_3}(X) \subset BN_{c_1}(X)$ and

hence $C_3 \Rightarrow C_1$

Example 5 : Let U={a, b, c, d, e} be the universe and C_1={{a}, {b, c}, {c, d}, {d, e}}
and C_2 = {{a}, {b, c, d},{b, d, e}, {e}}.

Clearly $C_1 \subseteq C_2$. Now for X = {a, c}, we get BN_{c_1}(X) = {b, c} $\supseteq$ {c} = BN_{c_2} (X) and for Y = {b, c}, we find BN_{c_1} (Y) = ϕ $\subseteq \{b, c, d\} = BN_{c_2}(Y)$

Thus neither $C_1 \Rightarrow C_2$ *nor* $C_2 \Rightarrow C_1$, that is, the two coverings are independent. Given coverings C_1 and C_2 of U, the covering C be

C = {{a}, {b,c},{c,d}, {d, e}, {e}, {b, c, d}, {b, d, e}] and the intersection and union of C_1 and C_2 be defined by
C_1U C_2 = {{a}, {e}, {b, c}, {c, d}, {d, e}} and C_1U C_2 = {{a}, {b, c, d}, {b, d, e, }}.

Clearly $C_1 \cap C_2 \Rightarrow C_1$ since for any subset X of U, $BN_{c_1 \cap c_2}(X) \Rightarrow BN_{c_1}(X)$ and $C_1 \cap C_2 \subseteq C_1$.

6. Other Types covering based Rough Sets

<u>**Definition** </u>**6.1** Let (U, C) be a covering approximation space. For $X \subseteq U$, the covering lower and upper appointments of X be defined by $\underline{apr}_c(X) = \underline{X} \cup A$, where

$\underline{X} = U\{K \in C : K \subseteq X\}$ *and*

$A = \cap\{Md(x) : x \in X\}$ if $\cap\{Md(x) : x \in X\} \subseteq X$

$= \phi$, otherwise

and

$\overline{apr}_c(X) = X^* = \cap\{K' : K \subseteq X'$ and $K \in X\}$

The approximation set X is called rough if $\underline{apr}_c(X) \neq \overline{apr}_c(X)$, otherwise X is called an exact set..

The set X is lower exact in the new type covering approximation space if X is the union of some elements of C and X is upper exact if X' is set the union of some elements of C.

Proposition 6.1 The new type covering lower and upper approximation have the following properties, for X, Y $\subseteq U$,

6.1 $\underline{apr}_c(X) \subseteq X, X \subseteq \overline{apr_c}(X)$

6.2 $\overline{apr}_c(\phi) = \phi, \underline{apr}_c(U) = U$

6.3 $\underline{apr}_c(\underline{apr}_c(X)) = \underline{apr}_c(X)$, $\overline{apr}_c(\overline{apr}_c(X)) = \overline{apr}_c(X)$

6.4 $X \subseteq Y \Rightarrow \overline{apr}_c(X) \subseteq \overline{apr}_c(Y)$

Next we will find two more improved covering spaces on a non empty finite universe U.

Definition 6.3 Let (U, C) be a covering approximation space. The generalized covering lower approximation and upper approximation for any set $X \subseteq U$ be defined as $\underline{apr}'_c(x) = \underline{X} \cup A$ and $\overline{apr'}_c(x) = \overline{X} \cap X^*$

The set X is called rough if an only if $\underline{apr}'_c(X) \neq \overline{apr'}_c(x)$The subset $X \subseteq U$ becomes exact in this generalized covering approximation if and only if X is the union of some elements of C and X' is the union of some elements of C.

7. Conclusion:

To sum up, tracing definition for new types of generalized covering based rough set with the lower approximation and upper approximation operators for covering based rough sets are improved. Some theorems are established for the comparison of other types of covering based rough set. The dependency on covering approximation spaces is expounded.

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